Effect of Modified Aerodynamic Strip Theories on Rotor Blade Aeroelastic Stability

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Various existing unsteady aerodynamic strip theories that have been developed in the past for both fixed and rotary wing aeroelastic analyses are modified to make them applicable to the coupled flap-lag-torsional aeroelastic problem of a rotor blade in hover. These corrections are primarily because of constant angle of attack, constant inflow, and variable freestream velocity due to lead-lag motion. Next, the modified strip theories are incorporated in a coupled flap-lag-torsional aeroelastic analysis of the rotor blade in hover, and the sensitivity of the results to aerodynamic assumptions is examined.

Nomenclature

a_i	= two-dimensional incompressible lift curve slope
$\bar{a}$	= offset between elastic axis and midchord, positive aft, nondimensional with respect to semichord
$[A(k,M,\bar{m},\bar{h}]$	= generalized complex aerodynamic matrix
b	= half-chord nondimensionalized with respect to R
$[B]$	= symbolic damping matrix used in discussion of complex eigenvalue problem
C_{d0}	= profile-drag coefficient
$C(k)$	= Theodorsen's lift deficiency function
$C'(k,\bar{h},\bar{m})$	= Loewy's modified lift deficiency function
$[D]$	= symbolic matrix used in discussion of complex eigenvalue problem
e_1	= blade pitch bearing offset
f_1	= generalized coordinate, first coupled root torsional mode
f_1^0	= static value of f_1 in hover
Δf_1	= perturbation in f_1 about f_1^0
g	= fictitious structural damping
g_1	= generalized coordinate, first normal flap mode
g_1^0	= static value of g_1 in hover
Δg_1	= perturbation in g_1 about g_1^0
h'	= wake spacing
Δh	= plunging harmonic motion used in unsteady aerodynamic theories ($\Delta\bar{h}$ complex constant quantity)
h_1	= generalized coordinate, first normal in-plane mode
h_1^0	= static value of h_1 in hover
Δh_1	= perturbation in h_1 about h_1^0
$\bar{h}$	= nondimensional wake spacing ($h'/br=\bar{h}$)
i	= $\sqrt{-1}$
I_b	= mass moment of inertia of elastic part of the blade about its root
k	= $\omega bR/\Omega(x_0+e_1)=\omega bR/V_0$ = reduced frequency
k_A	= polar radius of gyration of cross-sectional area effective in carrying tensile stresses about the elastic axis ($\bar{k}_A=k_A/\ell$)
k_ϕ	= root torsional spring constant, control system stiffness
k_0	= polar radius of gyration of cross-sectional mass about its center of gravity ($\bar{k}_0=k_0/\ell$)
ℓ	= length of blade capable of elastic deformation
L_{Y2},L_{Z2}	= aerodynamic load per unit length in lag and flap directions[5]
L	= two-dimensional unsteady lift per unit span, positive up
L_h,L_α	= incompressible unsteady aerodynamic coefficients
ℓ_h,ℓ_α	= general unsteady aerodynamic coefficients used for both compressible and incompressible cases
ℓ'_h,ℓ'_α	= unsteady aerodynamic coefficients used for compressible unsteady rotary wing aerodynamics
M_A	= unsteady pitching moment about elastic axis, positive, leading edge up, per unit span
M_h,M_α	= incompressible unsteady aerodynamic coefficients
m_h,m_α	= general unsteady aerodynamic coefficients used for both compressible and incompressible cases
m'_h,m'_α	= unsteady aerodynamic coefficients used for compressible unsteady rotary wing aerodynamics
M	= Mach number at a radial station
$\bar{m}=\omega/\Omega$	= frequency ratio
q_{xA}	= distributed aerodynamic torque about the x direction
q	= vector of generalized coordinates
R	= blade radius
t	= time
U_{Y2},U_{Z2}	= relative velocity components of blade elastic axis in the J_2 and K_2 directions[5]
u,v,w	= x,y, and z displacements of a point on the elastic axis of the blade
U_P,U_T	= velocity components approximately normal and parallel to the hub plane
U_{P0},U_{T0}	= constant part of U_P,U_T
$\Delta U_P,\Delta U_T$	= time-dependent harmonic part of U_P and U_T
V	= oncoming freestream velocity ($=U_T$)
V_0	= constant part of V
ΔV	= time-dependent harmonic part of V

Presented at the AIAA/ASME/SAE 17th Structures, Structural Dynamics, and Materials Conference, King of Prussia, Pa., May 5-7, 1976 (in bound volume of Conference papers, no paper number); submitted June 1, 1976; revision received March 17, 1977.

Index categories: Aeroelasticity and Hydroelasticity; Nonsteady Aerodynamics; Propeller and Rotor Systems.

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v_0 = constant part of v
Δv = time-dependent harmonic part of v
w_0 = constant part of w
Δw = harmonic, time-dependent part of w
$x_0 = x - e_1$ = spanwise coordinate for elastic part of the blade ($\bar{x}_0 = x_0/\ell$)
$x_A, (\bar{x}_A = x_A/bR)$ = blade cross-sectional aerodynamic center offset from elastic axis, shown in Fig. 1B, Ref. 5; positive for aerodynamic center before elastic axis
$x_I, (\bar{x}_I = x_I/\ell)$ = blade cross-sectional mass center of gravity offset from elastic axis[5]
Z = complex flutter parameter

Greek Symbols

$\Delta\alpha$ = harmonic pitching angle used in unsteady aerodynamic theory ($\overline{\Delta\alpha}$ constant complex part)
β_P = preconing, inclination of feathering axis with respect to hub plane
γ = lock number $\gamma = (2\rho_\infty bR^5 a_i / I_b)$
γ_1 = first in-plane bending mode shape
ϵ_D = symbolic quantity having the same order of magnitude as the elastic slopes
η_1 = first flapwise bending mode shape
η_{SF1}, η_{SL1} = viscous structural damping coefficients, in percent of critical damping, for first flap and lag mode, respectively
$\theta_G(\bar{x}_0)$ = total geometric pitch angle at a blade cross section, composed of built-in twist and collective pitch
θ_B = built-in twist
θ = collective pitch measured from $x\ y$ plane (hub plane)
λ = inflow ratio, induced velocity overdisk, positive down, nondimensionalized with respect to ΩR
ρ_∞ = density of air
σ = blade solidity ratio (blade area/disk area)
ϕ = total elastic torsional deformation
ϕ_1 = first coupled root-torsional mode
ϕ_0 = constant part of ϕ
$\Delta\phi$ = harmonic, time-dependent part of ϕ
ψ = azimuth angle of blade ($\psi = \Omega t$) measured from straight aft position
ω = flutter frequency ($\bar{\omega} = \omega/\Omega$)
$\bar{\omega}_{F1}, \bar{\omega}_{L1}$ = natural frequency of first flap or lag mode (rotating), nondimensionalized with respect to Ω
$\bar{\omega}_{T1}$ = natural frequency of first torsional mode (rotating), nondimensionalized with respect to Ω
Ω = speed of rotation

Superscripts

$(^*)$ = differentation with respect to ψ
$(\dot{\ })$ = differentiation with respect to time

I. Introduction and Statement of Problem

A CONSIDERABLE amount of work in the area of rotary wing aeroelasticity[1-6] has been devoted to the problem of adequate mathematical modeling of the aeroelastic behavior of hingeless rotor blades in hover or forward flight. In these studies, the main emphasis was on the modeling of the structural and inertia operators associated with this aeroelastic problem, whereas almost no work was done on determining the effect of unsteady aerodynamics on this problem.

In the past, various unsteady aerodynamic strip theories have been developed for both fixed and rotary wing applications.[7-11] The effect of unsteady aerodynamics has been known to affect both the vibration levels and the aeroelastic stability of rotor blades[12]; however, very little work has been done on systematic incorporation of unsteady aerodynamics into rotary wing aeroelastic analyses that include all three (flap, lag, and torsional) degrees of freedom, essential for this aeroelastic problem. A recent study by Anderson and Watts[13] has indicated that unsteady aerodynamics can affect the aeroelastic stability of a hingless rotor significantly. In Ref. 13, only Loewy's unsteady aerodynamic theory was used; however, the incorporation of the unsteady aerodynamic theory in the blade equations of motion was not done carefully. In particular, the unsteady aerodynamic coefficients given in Ref. 13 are not consistent with a rotor blade having flap, lag, and torsional degrees of freedom, which is the case considered in Ref. 13.

The present paper has three main objectives. First, it will be shown that the various unsteady aerodynamic strip theories[7-11] have to be modified and cast into a form that is applicable directly to the rotor blade aeroelastic problem in hovering flight when all three (flap, lag, and torsional) degrees of freedom are considered. The second objective of this paper is a systematic incorporation of the modified unsteady aerodynamic strip theories into a coupled flap-lag-torsional aeroelastic analysis of a hingeless blade, such as indicated in Ref. 5. The last objective is to determine the sensitivity of the aeroelastic stability boundaries to the incorporation of various unsteady aerodynamic strip theories, in the flutter analysis. All of the objectives just described have not been treated in the literature before.

II. Modified Unsteady Aerodynamic Strip Theories for Rotor Blade Flutter Calculations

A. Available Unsteady Strip Theories

The problems encountered when attempting to apply the available unsteady aerodynamic strip theories to rotor blade aeroelastic analyses can be understood best by reviewing briefly these theories, together with their inherent fundamental assumptions. The various theories that will be considered are 1) Theodorsen's incompressible fixed wing theory,[7,8] 2) Loewy's incompressible rotary wing theory,[9] 3) unsteady compressible fixed wing theory,[7,8] and 4) unsteady two-dimensional compressible rotary wing theory.[10,11] The geometry of the problem employed in formulating these theories is shown schematically in Fig. 1.

The assumptions commonly used in the various unsteady aerodynamic strip theories are as follows:

1) The cross section of the wing or blade is assumed to perform only simple harmonic pitching and plunging oscillations, as indicated in Fig. 1. These are

$$\Delta\dot{h} = \overline{\Delta h}\, e^{i\omega t}; \qquad \Delta\dot{\alpha} = \overline{\Delta\alpha}\, e^{i\omega t} \tag{1}$$

2) The velocity of the oncoming airflow is constant.

3) The airfoils are represented by vortex sheets or doublet sheets.

4) The wake vortices (or doublets) move with the fluid; i.e., if the undisturbed fluid is at rest, they keep their positions in space while the airfoil proceeds. The effects of vortex-vortex (doublet-doublet) interaction and viscous diffusion are neglected; i.e., vortex (or doublet) strength remains constant in time and space.

5) For theories that include compressibility, the flows are assumed to be subsonic.

6) The airfoil thickness and the amplitude of oscillation are assumed to be small compared to the chord.

7) The Kutta condition must be satisfied at the trailing edge.

The essential features of these theories are outlined briefly; additional details can be found in Ref. 15.

Theordosen's theory[7,8] is classical and needs no detailed description. The unsteady lift and moment can be written for

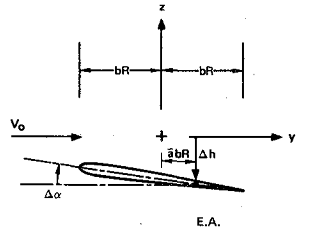

Fig. 1 Geometry of blade unsteady motion for conventional aerodynamics.

the case of simple harmonic motion as

$$L=\rho_\infty a_i V_0^2(k^2/2)bR\{L_h(\Delta h/bR)+[L_\alpha-(0.5+\bar{a})L_h]\Delta\alpha\} \tag{2}$$

where

$$L_h=-1+2(iV_0/b\omega R)C(k) \tag{3}$$

$$L_\alpha=-\tfrac{1}{2}+(iV_0/\omega bR)[1+2C(k)]+2(V_0^2/\omega^2b^2R^2)C(k) \tag{4}$$

$$M_A=(a_i/2)\rho_\infty V_0^2(bR)^2k^2\{[M_h+(\tfrac{1}{2}+\bar{a})L_h](\Delta h/bR)+[M_\alpha-(\tfrac{1}{2}+\bar{a})(-L_\alpha+M_h)-(\tfrac{1}{2}+\bar{a}^2)L_h]\Delta\alpha\} \tag{5}$$

where

$$M_h=\tfrac{1}{2};\qquad M_\alpha=\tfrac{3}{8}-(iV_0/\omega bR) \tag{6}$$

One of the convenient features of the unsteady lift and moment expressions in Theodorsen's theory is that it enables one to identify the circulatory and noncirculatory terms in the unsteady airloads. Furthermore, the effect of the unsteady wake always is represented by the terms that are multiplied by Theodorsen's lift deficiency function $C(k)$. It should be noted that noncirculatory acceleration terms, $\ddot{h}$ and $\ddot{\alpha}$, have been neglected in previous rotary wing aeroelastic analyses.[3,5,14]

Loewy's rotary wing unsteady aerodynamic theory[9] is an incompressible theory, somewhat similar to Theodorsen's, where the effect of the spiral returning wake, beneath a rotor, is taken into account approximately. In the theory, it is assumed that the wakes, infinite in number, lie in planes parallel to the disk and are separated by the wake spacing h', which is dictated by the induced inflow velocity and the period of rotor rotation. Loewy has shown that for this case the unsteady aerodynamic lift and moment can be written in a form identical to classical Theodorsen theory, except that the conventional lift deficiency function $C(k)$ should be replaced by a more complicated lift deficiency function, given by $C'(k,\bar{m},\bar{h})$.

It should be noted that Loewy's theory has been intended to be used primarily in studying unsteady effects on rotors operating at low inflows, as well as unsteady effects at larger inflows for cases when the oscillatory frequency is significantly higher than the rotational speed. Experimental evidence obtained by Ham et al.[17] shows clearly that Loewy's unsteady aerodynamics can affect rotor blade response levels significantly, primarily at low inflow ratio. Additional details can be found in Ref. 9, 15, and 16.

Compressible fixed wing aerodynamic theory[7,8] has been developed by Possio. This theory is basically an extension of Theodorsen's theory to the compressible case. The complete solution of the equations of motion of the fluid are obtained by superimposing elementary solutions for a sinusoidal pulsating doublet along the chord.

For this case, it is customary to express the unsteady airloads by

$$L=(a_i/2)\rho_\infty V_0^2bR[\ell_h(\Delta h/bR)+\ell_\alpha\Delta\alpha] \tag{7}$$

$$M_A=(a_i/2)\rho_\infty V_0^2(bR)^2[m_h(\Delta h/bR)+m_\alpha\Delta\alpha] \tag{8}$$

where ℓ_h, ℓ_α, m_h, and m_α are now implicit functions of the reduced frequency and the Mach number.

Compressible rotary wing aerodynamic theories[10,11] have been developed by Hammond and Pierce[10] and by Jones and Rao.[11] Jones and Rao's theory is an extension of Loewy's theory to include compressibility and is based upon the velocity potential. Hammond and Pierce's theory is a similar correction, except that it is based upon the acceleration potential. Although both theories yield similar results, Jones and Rao's theory is computationally more efficient, and therefore the results presented in this paper will be based on this theory.

The loads, expressed in the customary form used in flutter analysis, are given by

$$L=(a_i/2)\rho_\infty V_0^2bR[\ell_h'(\Delta h/bR)+\ell_\alpha'\Delta\alpha] \tag{9}$$

$$M_A=(a_i/2)\rho_\infty V_0^2(bR)^2[m_h'(\Delta h/bR)+m_\alpha'\Delta\alpha] \tag{10}$$

where ℓ_h', ℓ_α', m_h', and m_α' are implicit functions of the reduced frequency k, the nondimensional wake spacing $\bar{h}$, the frequency ratio $\bar{m}$, and the Mach number M. Additional details on these theories can be found in Refs. 10, 11, 15, and 16.

B. Modification of Unsteady Strip Theories

General

The basic difficulty encountered when attempting to apply the unsteady aerodynamic theories just reviewed to rotor blade aeroelastic calculations is primarily due to the fact that a rotor blade having flap, lag, and torsional degrees of freedom violates assumptions 1, 2, and, to a certain degree, 6 of Sec. II. A. The main differences between these assumptions and the real behavior of a rotor blade are as follows:

1) In addition to the constant velocity of oncoming flow, the blade also experiences a time-dependent velocity variation due to its lead-lag motion.

2) In addition to the harmonic variation in the angle of pitch, because of elastic torsional motion of the blade, a constant collective pitch setting also is imposed on the airfoil.

3) The plunging velocity of the airfoil, $\Delta\dot{h}$, not only is composed of a harmonic part associated with elastic flapping motion, but, in addition, has a constant velocity component due to the inflow through the rotor disk.

4) Blade deflections are not necessarily small when compared to the thickness of the airfoil.

These differences indicate that certain modifications of the existing unsteady strip theories are required before applying them to rotor blade flutter calculations. The modifications are described in the following sections.

Modification of Classical Theodorsen Theory

The theory is modified first so as to include the effects due to a steady-state value of collective pitch setting and harmonic variation in the oncoming airflow, and, finally, the effect of constant inflow also is included. Corrections to Theodorsen's theory which account for variation in oncoming flow velocity were developed first by Issaacs.[18,19] Isaacs' theory was modified further by Greenberg[20] to include the effect associated with constant angle of pitch. A careful examination of Greenberg's assumptions reveals that his theory is only an approximate one, because he has neglected the effect of fore and aft excursions of the blade, relative to the mean velocity, on the wake. The approximate nature of Greenberg's theory, which also has been noted in Ref. 21, is based on the assumption that the nondimensional in-plane velocity fluctuation is small compared to the reduced flutter frequency. It can be shown that this assumption is equivalent to $\epsilon_D << b$, which is satisfied only roughly for rotor blades. With Greenberg's modification, Theodorsen's expression for lift can be written in a modified form as

$$L=(a_i/2)\rho_\infty V_0^2k^2bR\{L_h(\Delta h/bR)+[L_\alpha-(1/2+\bar{a})L_h]\Delta\alpha\}$$
$$+(a_i/2)\rho_\infty(bR)^2\Delta\dot{V}\theta_0+a\rho_\infty\dot{V}_0^2bR\theta_0\{1$$
$$+(\Delta V/V_0)[1+C(k)]\}-a_i\rho_\infty Vb\,RU_{P0} \tag{11}$$

where θ_0 is the constant part of the angle of pitch imposed on the blade. In Eq. (11), the second and third terms are correction terms obtained by Greenberg; the second term is an additional apparent mass term, whereas the third term is one due to variation in oncoming flow velocity. The last term is a correction term due to constant inflow which has been added by the authors. Finally, it should be mentioned that in all unsteady aerodynamic expressions the value of 2π has been replaced by the two-dimensional lift curve slope a_i. Similarly, the moment about the elastic axis of the blade can be written as

$$M_A=(a_i/2)\rho_\infty V_0^2(bR)^2\{[M_h+(1/2+\bar{a})L_h](\Delta h/bR)$$
$$+[M_\alpha-(1/2+\bar{a})(-L_h+M_\alpha)-(1/2+\bar{a})^2L_h]\Delta\alpha\}$$
$$-(a_i/2)\rho_\infty\bar{a}\Delta\dot{V}\theta_0(bR)^3+a_i\rho_\infty V_0^2(bR)^2\theta_0(\bar{a}+1/2)\{1$$
$$+(\Delta V/V_0)[1+C(k)]\}-(a_i/2)\rho_\infty V(bR)^2(\bar{a}+1/2)U_{P0} \tag{12}$$

The last term in Eq. (12) represents a contribution due to constant inflow.[15]

To facilitate the algebraic manipulation of Eq. (11) and (12), the following notation is introduced:

$$L=(a_i/2)\rho_\infty V_0^2bR[\ell_h(\Delta h/bR)+\ell_\alpha\Delta\alpha]$$
$$+(a_i/2)\rho_\infty(bR)^2\Delta\dot{V}\theta_0+a_i\rho_\infty V_0^2bR\theta_0\{1$$
$$+(\Delta V/V_0)[1+C(k)]\}-a_i\rho_\infty V\,bRU_{P0} \tag{13}$$

$$M_A=(a_i/2)\rho_\infty V_0^2(bR)^2[m_h(\Delta h/bR)+m_\alpha\Delta\alpha]$$
$$-(a_i/2)\rho_\infty\Delta\dot{V}\theta_0(bR)^3\bar{a}+a_i\rho_\infty V_0^2(bR)^2\theta_0(\bar{a}+1/2)\{1$$
$$+(\Delta V/V_0)[1+C(k)]\}-a_i\rho_\infty V(bR)^2(1/2+\bar{a})U_{P0} \tag{14}$$

where

$$\ell_h=k^2L_h \tag{15}$$

$$\ell_\alpha=k^2[L_\alpha-(1/2+\bar{a})L_h] \tag{16}$$

$$m_h=k^2[M_h+(1/2+\bar{a})L_h] \tag{17}$$

$$m_\alpha=k^2[M_\alpha-(1/2+\bar{a})(-L_h+M_h)-(1/2+\bar{a})^2L_h] \tag{18}$$

Equations (13) and (14), which contain modifications accounting for in-plane lead-lag motion, constant angle of pitch, and constant inflow, still are not in the proper form for the aeroelastic analysis of a helicopter rotor blade having flap, lead-lag, and torsional degrees of freedom. Additional modifications are required because in rotary wing aeroelasticity one has to assume that the slopes of the deformed blade are moderately large. This usually leads to dynamic stability equations that are linearized about a static equilibrium position, which is denoted by adding zero subscripts on the variables, as shown in Ref. 3-5 and 14. The displacement field of a rotor can be written, in general, as[5]

$$w=w_0+\Delta w=w_0+\Delta\bar{w}e^{i\omega t} \tag{19a}$$

$$v=v_0+\Delta v=v_0+\Delta\bar{v}e^{i\omega t} \tag{19b}$$

$$\phi=\phi_0+\Delta\phi=\phi_0+\Delta\bar{\phi}e^{i\omega t} \tag{19c}$$

where the last term in these expressions represents the simple harmonic motion corresponding to the flutter condition. In rotary wing aeroelasticity, it is customary to deal with the velocities of a point on the elastic axis of the blade. For this point, it can be shown that the velocities are related to the displacement field by[5,14,15]

$$U_P=-U_{Z2}=\frac{\partial w}{\partial x_0}v\Omega+\overset{*}{w}\Omega+R\Omega\lambda=U_{P0}+\Delta U_p \tag{20a}$$

$$U_T=-U_{Y2}=\overset{*}{v}\Omega+\Omega(x_0+e_I)=U_{T0}+\Delta U_T \tag{20b}$$

Combining Eqs. (19) and (20), it can be shown that

$$U_{P0}=\Omega R\lambda+\Omega\frac{\partial w_0}{\partial x_0}v_0 \tag{21}$$

$$\Delta U_P=\frac{\partial w_0}{\partial x_0}\Delta v\Omega+\frac{\partial\Delta w}{\partial x_0}\Omega v_0+\Delta\overset{*}{w}\Omega \tag{22}$$

$$U_{T0}=\Omega(x_0+e_I)=V_0 \tag{23}$$

$$\Delta U_T=\Delta\overset{*}{v}\Omega=\Delta V \tag{24}$$

$$\theta_0=\theta_G+\phi_0; \qquad \Delta\theta=\Delta\phi \tag{25}$$

Next, it is important to note that, when applying unsteady aerodynamics to rotary wing aeroelasticity, the quantity Δh in Eqs. (13) and (14) has to be identified as

$$\Delta\dot{h}=-\Delta U_p \tag{26}$$

Furthermore,

$$\frac{\Delta h}{bR}=\frac{1}{i\omega}\frac{\Delta\dot{h}}{bR}=\frac{1}{ki}\frac{\Delta\dot{h}}{\Omega(x_0+e_I)} \tag{27}$$

where $k=\omega bR/\Omega(x_0+e_I)$ is the local reduced frequency. From the geometry,[5]

$$\bar{x}_A=x_A/bR=(1/2+\bar{a}) \tag{28}$$

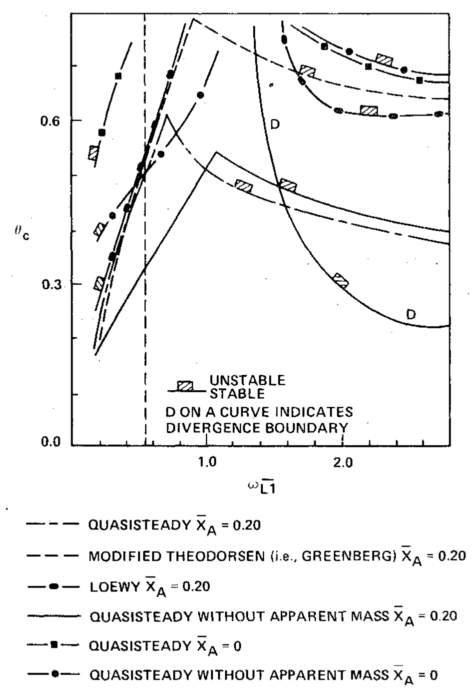

Fig. 2 Effect of modified classical unsteady aerodynamics on typical cases ($\bar{\omega}_{FI}=1.14019$, $\bar{\omega}_{TI}=6.17108$, and approximate, linearized, static equilibrium).

Substituting Eqs. (21-28) into Eqs. (13) and (17), one obtains the modified unsteady aerodynamic expressions which are valid for rotary wing applications:

$$L \cong L_{Z2} = \frac{a_i}{2}\rho_\infty bR\Big\{\Big[-\Omega^2(x_0+e_1)\Big(\frac{\ell_h}{ki}\Big)\Big(\Delta V\frac{\partial w_0}{\partial x_0} + v_0\frac{\partial \Delta w}{\partial x_0} + \bar{\omega}\Delta wi\Big) + \Omega^2(x_0+e_1)^2\ell_\alpha\Delta\phi\Big]$$
$$- k\Omega^2(x_0+e_1)\bar{\omega}(\theta_G+\phi_0)\Delta v + 2\Omega^2(x_0+e_1)^2(\theta_G+\phi_0) \times\Big(1+\frac{\bar{\omega}\Delta v}{x_0+e_1}[1+C(k)]\Big)$$
$$-2\Omega^2[(x_0+0_1)+\bar{\omega}i\Delta v]\Big[v_0\frac{\partial w_0}{\partial x_0}+\lambda R\Big]\Big\} \tag{29}$$

Similarly,

$$M_A = q_{xA} = \frac{a_i}{2}\rho_\infty(bR)^2\Big\{\Omega^2(x_0+e_1)^2\Big[-\Big(\frac{m_h}{ki}\Big)\frac{1}{x_0+e_1} \times\Big(\Delta v\frac{\partial w_0}{\partial x_0}+v_0\frac{\partial\Delta w}{\partial x_0}+\bar{\omega}i\Delta w\Big)+m_\alpha\Delta\phi\Big]$$
$$+k(x_0+e_1)\Omega^2\Delta v\bar{w}(\bar{x}_A-0.5)(\theta_G+\phi_0)$$
$$+2\Omega^2(x_0+e_1)^2(\theta_G+\phi_0)\bar{x}_A\Big(1+\frac{\bar{\omega}i\Delta v}{(x_0+e_1)}[1+C(k)]\Big)$$
$$-2\bar{x}_A\Omega^2[(x_0+e_1)+i\bar{\omega}\Delta v]\Big[v_0\frac{\partial w_0}{\partial x_0}+R\lambda\Big]\Big\} \tag{30}$$

In addition to the distributed lift and torsional moment, an expression for the aerodynamic load in the chordwise direction also is required for rotary wing aeroelastic applications. This expression can be obtained using the approximate method given in Ref. 5 and 14; thus,

$$L_{Y2} = -L_{Z2}\Big(\frac{U_p}{U_T}\Big) - \rho_\infty bR\,U_T^2C_{d0}$$
$$= \frac{-L_{Z2}}{(x_0+e_1)}\Big\{\Big[v_0\frac{\partial w_0}{\partial x_0}+R\lambda+v_0\frac{\partial\Delta w}{\partial x_0} + \Delta v\frac{\partial w_0}{\partial x_0}+\bar{\omega}i\Delta w\Big] - \Big[\bar{\omega}i\Delta v\frac{v_0(\partial w_0/\partial x_0)+R\lambda}{(x_0+e_1)^2}\Big]\Big\}$$
$$-\rho_\infty bR\,C_{d0}\Omega^2[(x_0+e_1)^2+2(x_0+e_1)i\bar{\omega}\Delta v] \tag{31}$$

Modification of Loewy's Rotary Wing Unsteady Theory

The modified version of Loewy's unsteady aerodynamic theory can be obtained directly from Eq. (29-31) when $C(k)$ in these expressions is replaced by Loewy's modified lift deficiency function $C'(k, \bar{h}, \bar{m})$. Furthermore, it is understood that in this case the unsteady aerodynamic coefficients $\ell_h(k,\bar{h},\bar{m})$, $\ell_\alpha(k,\bar{h},\bar{m})$, $m_h(k,\bar{h},\bar{m})$, and $m_\alpha(k,\bar{h},\bar{m})$ also will be functions of the reduced frequency, wake spacing, and frequency ratio.

Modification of the Compressible Fixed Wing Unsteady Aerodynamic Theory

For the case of compressible unsteady aerodynamics, the modifications required have to be made in a different manner. Again, the modified theory can be obtained by using Eqs. (29-31) as starting point. For this case, the unsteady aerodynamic coefficients $\ell_h(k,M)$, $\ell_\alpha(k,m)$, $m_h(k,M)$, and $m_\alpha(k,M)$ are evaluated from a fixed wing unsteady aerodynamic theory such as Possio's[7,8]; thus, these coefficients now will depend on the local Mach number M at a certain radial station.

The last three terms in Eqs. (29) and (30), representing, respectively, the noncirculatory or apparent mass term, variable oncoming velocity, and the effect of constant inflow, are modified in the following manner. The noncirculatory forces are associated with the pumping action of the wing during oscillation and, hence, are functions of the oscillatory velocity component perpendicular to the wing surface and not of freestream velocity directly, as is the case for the circulatory terms. Therefore, the effect of compressibility on the noncirculatory term is ignored,[22] and this term is left unchanged. The last two terms are modified following a method suggested by Garrick[8] which also has been used by Yates.[22] This procedure is based on the assumption that the direction and the phase lag of the unsteady aerodynamic forces remain unaffected by compressibility. Consequently, the last two terms of Eq. (29) and (30) are corrected so as to reflect the change of the incompressible two-dimensional lift curve slope with Mach numbers, according to the Prandtl-Glauert rule. The final form of the modified version of Eq. (29) can be written as

$$L \cong L_{Z2} = \frac{a_i}{2}\rho_\infty bR\Big\{\Big[\Omega^2(x_0+e_1)\Big(\frac{\ell_h}{ki}\Big)\Big(\Delta v\frac{\partial w_0}{\partial x_0} + v_0\frac{\partial w}{\partial x_0}+\bar{\omega}\Delta wi\Big)+\Omega^2(x_0+e_1)^2\ell_\alpha\Delta\phi\Big]$$
$$-k\Omega^2(x_0+e_1)^2\bar{\omega}(\theta_G+\phi_0)\Delta v$$
$$+\frac{2\Omega^2(x_0+e_1)^2(\theta_G+\phi_0)}{\sqrt{1-M^2}}\Big(1+\frac{\bar{\omega}i\Delta v}{(x_0+e_1)}[1+C(k)]\Big)$$
$$-\frac{2\Omega^2}{\sqrt{1-M^2}}[(x_0+e_1)+\bar{\omega}i\Delta v]\Big[v_0\frac{\partial w_0}{\partial x_0}+R\lambda\Big]\Big\} \tag{32}$$

and a similar correction is made to Eq. (30). Equation (31) is left unchanged.

Modification of Compressible Rotary Wing Unsteady Aerodynamic Theories

The modified theories can be obtained from Eq. (32) and the similarly modified moment equation [Eq. (30)] by replacing ℓ_h, ℓ_α, m_h, and m_α in these equations by ℓ_h', ℓ_α', m_h', and m_α', where these quantities are evaluated from a compressible, unsteady, rotary wing aerodynamic theory,[10,11] where it should be understood that the unsteady aerodynamic coefficients $\ell_h'(k,M,\bar{h},\bar{m})$, $\ell_\alpha'(k,M,\bar{h},\bar{m})$, $m_h'(k,M,\bar{h},\bar{m})$, and $m_\alpha'(k,M,\bar{h},\bar{m})$ are now functions of the wake spacing and frequency ratio, in addition to being dependent upon the reduced frequency and the local Mach number. Furthermore, in Eq. (32) and in the corresponding term in the moment equation $C(k)$ should be replaced by $C'(k,\bar{h},\bar{m})$.

III. Aeroelastic Analysis Using the Modified Strip Theories

A. Brief Review of the Formulation of the Rotor Blade Aeroelastic Equations

A description of the basic assumptions, together with a brief derivation of the structural and inertial operators associated with the coupled flap-lag-torsional aeroelastic problems, has been presented in Ref. 5. The reduction of the partial differential equations to ordinary differential form and linearization of the equations also can be found in Ref. 5. These also are presented with considerable detail in Ref. 15. Therefore, the main purpose of this section is the identification of the similarities and the main differences in these items when compared with Ref. 5.

The basic assumptions used in this study are similar to those presented in Sec. II B of Ref. 5. However, the assumptions regarding the aerodynamic loads are completely different. The modified unsteady aerodynamic strip theories described in the previous sections replace the simple aerodynamic operator, which has been used previously.[5,14]

One of the most important assumptions made in Ref. 5 and retained in the present study is the assumption of moderate blade deflections, resulting in small strain and moderate blade slopes. Thus,

$$\frac{\partial w}{\partial x_0} = \frac{\partial v}{\partial x_0} = \phi = O(\epsilon_D); \qquad \frac{\partial}{\partial x_0} = \frac{\partial}{\partial \psi} = O(1)$$

Furthermore, it is assumed that terms of $O(\epsilon_D^2)$ are negligible compared to terms of $O(1)$. Based on these assumptions, an ordering scheme is developed which is used subsequently to simplify the equations.[5,15,16]

B. Equations of Motion and Determination of Flutter Boundaries

Using the assumptions and the ordering scheme, the equations of motion are derived in general, partial nonlinear differential form.[5,15] The structural and inertia operators are similar to those in Ref. 5, whereas the aerodynamic operators are represented by Eqs. (29-32). It should be emphasized that the aerodynamic operators, as presented in this study, are already in a linearized partial differential form.

The system of general, coupled, partial differential equations of motion presented in the previous section is transformed into a system of ordinary nonlinear differential equations by using Galerkin's method to eliminate the spatial variable. In this process, the elastic degree of freedom in the problem is represented by the uncoupled, free-vibration modes of a rotating blade. The present study is restricted to the case of a single elastic mode representing each elastic degree of freedom. The general case, with an arbitrary number of modes, is treated in Ref. 15. The modal representation of the elastic degrees of freedom is given by

$$w_e = \ell\eta_1(\bar{x}_0)g_1(\psi) \tag{33a}$$

$$v_e = -\ell\gamma_1(\bar{x}_0)h_1(\psi) \tag{33b}$$

$$\phi = \phi_1(\bar{x}_0)f_1(\psi) \tag{33c}$$

When performing the spanwise integrations required in the application of Galerkin's method, the unsteady aerodynamic loadings will be functions of the spanwise station because $\ell_h'(k,M,\bar{h},\bar{m})$, where $k=\omega bR/\Omega(x_0+e_1)$ and $M(x_0)=\Omega(x_0+e_1)/a_\infty$, where a_∞ is the speed of sound, in undisturbed flow. If exactly performed, this integration would introduce a complication deemed unnecessary for the trend-type study attempted in this paper. Results obtained by Zvara[24] indicated that good accuracy can be obtained when, instead of exact integration, the unsteady aerodynamic coefficients are based upon k evaluated at $0.8R$. At this point, it also was assumed that the trends associated with the compressibility also will be represented in a reasonable manner by using the value of the Mach number at $0.8R$. Therefore, all of the unsteady aerodynamic coefficients used in this study are based upon the values of k and M evaluated at $0.8R$.

The resulting nonlinear, ordinary differential equations are linearized about a suitable equilibrium position. The process of linearization consists of expressing the elastic part of the displacement field as

$$w_e = \eta_1(g_1^0 + \Delta g_1) \tag{34a}$$

$$v_e = -\gamma_1(h_1^0 + \Delta h_1) \tag{34b}$$

$$\phi = \phi_1(f_1^0 + \Delta f_1) \tag{34c}$$

The static equilibrium position is represented by the static values of the generalized coordinates g_1^0, h_1^0, and f_1^0; as pointed out in Ref. 5, there are two possible equilibrium positions that can be calculated: 1) an *approximate* static equilibrium position obtained by neglecting nonlinear combinations in the quantities g_1^0, h_1^0 and f_1^0; or 2) an *exact* static equilibrium position, obtained from the solution of the nonlinear algebraic equations defining the equilibrium position. Both methods were used in Ref. 15, where additional details can be found; however, the majority of the results are based on the approximate static equilibrium position.

The linearized equations of motion can be written in the following generalized form:

$$[M]\{\ddot{q}\} + [B]\{\dot{q}\} + ([\bar{M}_i\bar{\omega}_i^2] + [A(k,M,\bar{m},\bar{h})])\{q\} = 0 \tag{35}$$

where $[M]$, $[B]$, and $[A(k,M,\bar{m},\bar{h})]$ are square matrices having an order equal to the number of modes used in the analysis. The elements of $[M]$ and $[B]$ are, in general, real, whereas $[A(k,M,\bar{m},\bar{h})]$ has complex elements when modified unsteady aerodynamic strip theories are used. In Eq. (35), $[M]$ represents the mass matrix, M_i are the generalized masses, and $\{q\}^T = \lfloor \Delta g_1 \Delta h_1 \Delta f_1 \rfloor$.

It is important to note that the conventional $V-g$ method of flutter analysis used in fixed wing aeroelasticity is not directly applicable to the rotary wing aeroelastic problem because the collective pitch setting θ is the important flutter parameter for this problem, instead of the velocity V used in the fixed wing problem. Fortunately, a somewhat similar approach can be used in this problem also.

First an artificial damping coefficient denoted by g is introduced and combined with the rotating natural frequency of the ith degree of freedom ω_i in the form $\omega_i^2(1+ig)$; g also is combined with the flutter frequency ω to yield a complex eigenvalue Z, defined by

$$Z = (1/\bar{\omega}^2)(1+ig) = X + iY \tag{36}$$

In the usual manner, at the flutter condition, simple harmonic motion is stipulated by assuming

$$\{q\} = \{\bar{q}\}e^{i\omega\psi} \tag{37}$$

Combination of Eqs. (35-37) yields

$$\Big(-[M]\bar{\omega}^2 + i\bar{\omega}[B] + [A(k,M,\bar{m},\bar{h})] + \big[{}^{\diagdown} M_i\bar{\omega}_i^2\bar{\omega}^2 M_{\diagdown}\big]\Big)\{q\} = 0 \tag{38}$$

which can be reduced further to the complex eigenvalue form

$$([D] - Z[I])\{q\} = 0 \tag{39}$$

The solution to the complex eigenvalue problem given by Eq. (39) yields a number of complex eigenvalues equal to the number of modes employed in the aeroelastic analysis. For each complex eigenvalue ${}^iZ$, one has

$${}^iX = 1/{}^i\bar{\omega}^2; \qquad {}^ig = {}^i\bar{\omega}^2\,{}^iY \tag{40}$$

where i denotes the modal number. The system crosses from the stable region into the unstable region whenever one of the ${}^ig=0$, for $i=1,2,\ldots,n$, or whenever the artificial damping changes from negative to positive.

It should be noted that the situation is complicated further by the fact that the aerodynamic matrix, which is a function of k becomes related directly to the unknown real part of the complex eigenvalue ${}^iX$ by virtue of $k=\omega bR/(\Omega R\ 0.8)=\omega b$

1.25. Since $\bar{\omega}$ is unknown until the complex eigenvalue is obtained, an iterative process must be used to insure that k (and $\bar{m}$), used to evaluate the unsteady aerodynamic coefficients, is equal to $\bar{\omega}$ obtained from Eq. (40), ${}^i\bar{\omega}=1/X$. The method used to perform this iteration is somewhat similar to the one described by Hassig.[23] In reality, the iterative scheme used is a double iteration; in the first stage $\bar{\omega}$ is iterated upon, whereas in the second stage the collective pitch angle θ is varied until ${}^ig=0$ for one mode. Additional details on the iterative procedure can be found in Refs. 15 and 16.

IV. Results and Discussion

A. Information for the Computation of Stability Boundaries

The numerical results obtained in this study were computed using the same simplifying assumptions and inflow relation as given in Ref. 5. The unsteady aerodynamic coefficients needed for computation of the unsteady aerodynamic loads were obtained using a computer program that was developed by C.E. Hammond, Langley Directorate, U.S. Army Air Mobility Research and Development Laboratory, NASA Langley Research Center. A specially modified version of this program was provided by Hammond and incorporated by the authors into the aeroelastic analysis program.

In all of the computations, the following numerical values were used:

$$C_{d0}=0.01; \quad a_i=2\pi; \quad \sigma=0.08$$

$$\gamma=8.0; \quad x_I=0; \quad e_I=0$$

$$b=0.0313; \quad \bar{k}_0=\bar{k}_A=0.02$$

$$\beta_P=\eta_{SFI}=\eta_{SLI}=0.0$$

Also, as indicated in Sec. III B, k and M are evaluated at the blade spanwise station $x_0=0.8R$. Additional details can be found in Ref. 15.

B. Results

The main purpose of this study is to examine the sensitivity of the coupled flap-lag-torsional aeroelastic stability boundaries of a single blade, in hover, to the aerodynamic assumptions used. For this reason, the values of $\bar{\omega}_{FI}$ and $\bar{\omega}_{TI}$ were chosen so as to be representative of typical hingeless blade configurations and were not changed during the sensitivity study. However, the lead-lag frequency was varied over a reasonably wide range, $0.2<\bar{\omega}_{LI}<2.5$, representing a relatively wide class of possible blade configurations.

The results presented in Fig. 2 show the effect of the various incompressible, modified, unsteady aerodynamic theories on the flutter boundaries, in the presence of an offset between the aerodynamic center and the elastic axis. Each stability boundary shown consists basically of two branches: 1) the left-hand-side branch, corresponding to lower values of $\bar{\omega}_{LI}$, which usually is denoted the flap-lag branch, because the flutter frequency on this branch is close to the lead-lag frequency; and 2) a right-hand-side branch, corresponding to higher values of $\bar{\omega}_{LI}$, in which the torsional degree of freedom plays the important role. The flutter frequency on this branch is usually somewhere between the first flap and first torsional frequency (when one rotating mode is used to represent each elastic degree of freedom). Furthermore, it should be noted that the vertical dashed lines in Figs. 2-4 denote the matched stiffness configuration, for which the structural stiffnesses in the flapwise and lagwise directions, respectively, are equal. Under perfectly symmetric conditions, $\bar{x}_A=0$, no flap-lag type of instability should be encountered with these configurations; however, it has been pointed out[5] that this situation is highly dependent upon a number of small nonlinear terms defining the static equilibrium of the blade (associated with both aerodynamic and inertia terms). When the approximate, linearized, static equilibrium position is used, and the aerodynamic terms are linearized in partial differential form in order to accommodate the various modified unsteady theories, the flap-lag stability boundary can intersect the matched stiffness line at very high values of θ_c, $\theta_c>0.50$. These excessive values of θ_c are beyond the validity of the aerodynamic operators and the ordering scheme used. Furthermore, they exceed the practical operating conditions of the blade, and, therefore, one should not attach too much significance to this item.

The lines denoted by the captions "quasisteady without apparent mass" are representative of previous results,[5] where noncirculatory terms, except damping in pitch, were neglected, and $C(k)$ was assumed to be $C(k)=1$. As shown in Fig. 2, with $\bar{x}_A=0$, the blade is quite stable, and the values of critical pitch setting θ_c above which the blade becomes unstable are relatively high. Introducing a destabilizing offset, $\bar{x}_A=0.20$, between the aerodynamic center and the elastic axis reduces considerably the values of θ_c at which instabilities can occur. Unsteady aerodynamics are known to be significant at lower inflows and, therefore, lower pitch settings; thus, configurations that include an $\bar{x}_A$ offset are more

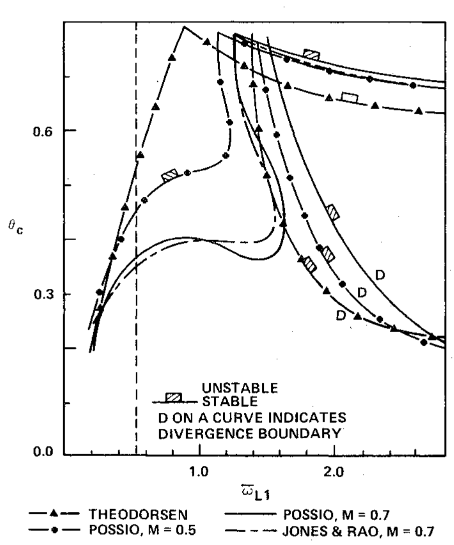

Fig. 3 Effect of modified compressible unsteady aerodynamics on typical cases ($\bar{\omega}_{FI}=1.14209$, $\bar{\omega}_{TI}=6.17018$, $\bar{x}_A=0.20$, andapproximate, linearized, static equilibrium).

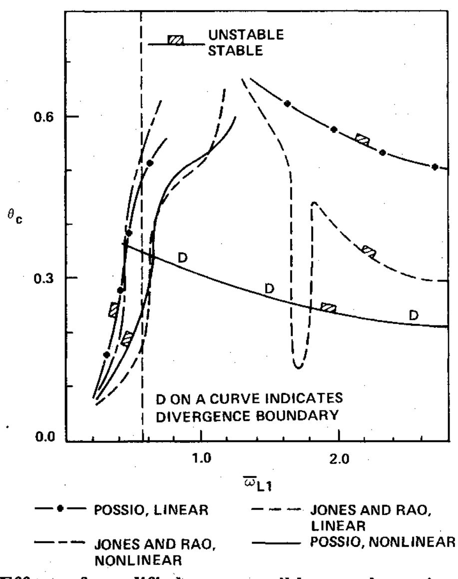

Fig. 4 Effect of modified compressible aerodynamics on typical cases, with two modes in each degree of freedom ($\bar{\omega}_{FI}=1.14029$, $\bar{\omega}_{TI}=6.17018$, $M=0.5$, $\bar{x}_A=0.20$, and linear or nonlinear static equilibrium).

suitable for studying the sensitivity of the boundaries to unsteady aerodynamics.

The effect of including the noncirculatory (or apparent mass) terms, yielding a consistent quasisteady case, is evident by comparing the appropriate boundaries in Fig. 2 with those having the caption "without apparent mass." Although the effect on noncirculatory terms is minor on the flap-pitch branch, its effect on the flap-lag branch is considerable. This is because of the known sensitivity of the flap-lag branch to small second-order terms, compared to which the noncirculatory terms represent first-order terms. It also is interesting to note that the flap-lag branch seems to be more stable when all apparent mass terms are included.

The effect of modified classical unsteady aerodynamics on a typical stability boundary is illustrated in Fig. 2 by the boundary having the caption "modified Theodorsen." As indicated in the figure, the unsteady aerodynamics primarily affect the flap-pitch branch of the stability boundary. This is because the flutter frequency is much higher on this branch, and, therefore, it is sensitive to unsteady aerodynamics. It also is evident that the flap-lag branch is insensitive to unsteady aerodynamics; however, it is sensitive to the noncirculatory (or apparent mass) terms. From the plot, it also is clear that, when neglecting these terms, as is commonly done in rotary wing aeroelasticity,[3-5] the blade appears less stable than it could be in reality. However, it obviously represents a conservative assumption for the cases considered.

The effect of Loewy's modified unsteady aerodynamics on a typical stability boundary also is shown in Fig. 2. The effect of this theory on the flap-lag branch is negligible; however, the effect on the flap-pitch branch is noticeable. The main difference between Loewy's unsteady aerodynamics and classical Theodorsen theory is the effect of the returning wake. For the high inflow ratios associated with the values of θ_c shown in the figure, the effect of the returning wake is minor because it is being removed by the high inflow.

It should be mentioned that the results presented in Figs. 2 and 3 are based upon the approximate, linearized, static equilibrium position of the blade.[5,15] However, the divergence boundaries[5,15] shown in these figures are nonlinear divergence boundaries.[5] It is interesting to note that the nonlinear divergence boundary is more critical than the flutter boundary, for the flap-pitch branch, at higher values of $\bar{\omega}_{L1}$.

The effect of modified compressible unsteady aerodynamics on the typical cases is illustrated in Fig. 3. Consider first the fixed wing type of compressible theory (Possio's); as shown, the effect of compressiblity on the flap-lag branch is considerable, whereas the flap-pitch branch has only a limited sensitivity to compressibility in the range of lead-lag frequencies shown in the plot.

It is worthwhile re-emphasizing that the Mach number used is a representative Mach number taken at $0.8R$. An additional detail regarding Fig. 3 is relevant. As shown, the lines for $M = 0.5$ start from above the incompressible line at low values of $\bar{\omega}_{L1}$, whereas the line representing $M = 0.7$ starts from below. This behavior is consistent with the results presented for low frequencies in Ref. 10.

A comparison of the effects of modified, compressible, unsteady fixed wing and modified, compressible, unsteady rotary wing aerodynamics is presented in Fig. 3. The difference between the two curves is due to the effects of the returning wake. Wake effects can be expected to be important when inflow ratios are low and frequencies relatively high. On the flap-pitch branch, the frequency is relatively high; however, the inflow ratio also is very high, and, therefore, the difference between the two curves is small. Between $0.9 < \bar{\omega}_{L1} < 1.4$, the frequency is much lower because this part of the stability boundary is the flap-lag branch. The inflow ratio, however, also is smaller, and the difference between the two curves is visible. It also is interesting to note the oscillation of the curve marked "Jones and Rao" about the fixed wing compressible curve. This oscillation, which occurs at integer multiples of the frequency ratio of the equivalent blade, is a typical characteristic of the unsteady aerodynamic theories, which have in them the effect of returning wake.

Again, it should be noted that the divergence boundaries can be more critical than the flap-pitch flutter boundaries. The single point, marked by a cross in Fig. 3, represents a calculation based on Hammond's unsteady theory[10]; the result is very close to Jones and Rao's[9] theory. However, it takes considerably longer to compute, and, therefore, Jones and Rao's theory was used primarily to achieve significant savings in computing time.

Figure 4 presents some additional results, showing the sensitivity of the boundaries to the number of modes and the static equilibrium position used in the analysis. Two rotating modes are used for each elastic degree of freedom. The flap-lag branch exhibits considerable sensitivity to the incorporation of the nonlinear equilibrium position. However, the most interesting result in Fig. 4 is the narrow, unstable region occuring with Jones and Rao's theory which dips slightly below the divergence boundary. These regions[15] were obtained only with unsteady theories containing the effect of the returning wake and with a multimode-type analysis. From a qualitative point of view, they resemble a type of instability encountered in Ref. 13. In this region, the second flap mode appears to interact with the first torsional mode; somewhat similar results were obtained[15] with Loewy's modified theory also. From the additional results presented in Ref. 15, it seems that using the quasisteady assumption; as commonly is done in rotary wing aeroelasticity, usually will yield conservative stability boundaries; thus, the practical value of the narrow unstable region in Fig. 4 may not be very significant. A considerable amount of additional results can be found in Ref. 15 which clearly cannot be included in a short paper such as this.

V. Conclusions

The main conclusions obtained in the present study are summarized below. They should be considered to be indicative of trends, and their application to the design of a helicopter blade should be limited by the assumptions presented herein. These conclusions are based on both the limited number of results presented here and the additional results given in Ref. 15:

1) It was shown that the various unsteady aerodynamic strip theories that have been developed in the past have to be modified or reinterpreted when applying them to the coupled flap-lag-torsional aeroelastic problem of a rotor blade in hover. These modifications are caused primarily by constant angle of attack, constant inflow, and variable freestream velocity due to lead-lag motion.

2) The effect of the noncirculatory or apparent mass terms and the effect of compressibility seem to be important for the range of lead-lag frequencies in which most helicopter rotor blades are built, and it is recommended that these terms be included in aeroelastic analyses. However, it also should be noted that the results obtained in this study indicate that neglecting apparent mass terms is a conservative assumption, whereas neglecting compressibility is a nonconservative assumption.

3) Blade behavior seems to be sensitive to the number of modes and the type of static equilibrium position about which the equations are linearized. When unsteady aerodynamics are used, at least two modes for each elastic degree of freedom should be used, and the static equilibrium position should be obtained by solving the fully nonlinear, static equilibrium position.

4) Conclusions based upon aeroelastic analyses, using unsteady aerodynamics and a simplified flap-pitch model for the rotor blade, may not be valid when dealing with the "real" aeroelastic problem of a rotor blade having flap, lag, and torsional degrees of freedom.

Acknowledgment

This work was supported by the Langley Directorate, U.S. Army Air Mobility Research and Development Laboratory and NASA Langley Research Center, Hampton, Va., under NASA NGR 05-007-414. The help and advice of C.E. Hammond, Head of the Rotor Dynamics Section, U.S. Army Air Mobility Research and Development Laboratory, NASA Langley Research Center, who provided the authors with a modified version of his computer program which was used for calculating the unsteady aerodynamic coefficients, are acknowledged gratefully.

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